Analysis of Counter-Current Imbibition Including Gravity Force through Finite Difference Scheme

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Abstract

Spontaneous counter-current imbibition is one of the most important crude oil recovery processes in water-wet fractured reservoirs with low matrix permeability. This paper presents a numerical modeling of imbibition process when water is imbibed by capillarity and gravity forces in to an oil saturated vertical cube core to examine the effect of gravity force on spontaneous imbibition. In this modeling, it is assumed that imbibition is a diffusion process. Finite difference implicit method was used to solve the spontaneous imbibition equations. Accuracy of the modeling is investigated with comparison of the modeling results and the experimental data.

Keywords: Spontaneous imbibition, Counter-current imbibition, Oil recovery, Gravity force.

1. INTRODUCTION

Many attempts have been done to examine the imbibition's effective parameters. The fluids flow is complex in the fracture reservoirs, as the fluid can flow in the porous medium, that has low transmissibility and high capacity, and the narrow fracture, that has high transmissibility and low capacity. When the matrix is saturated with non-wetting phase and wetting-phase is in the fracture, the capillary and gravity forces cause displacement of wetting-phase into the porous medium, this process is called imbibition [1]. The imbibition process can be classified into two categories: co-current and counter-current flows. In the counter-current imbibition, oil and water flow in the opposite directions; in the co-current imbibition, water and oil flow in the same direction [2]. Blair (1964) modeled numerically the 1-dimentional counter-current imbibition process in a porous network and concluded that the rate of imbibition is sensitive to capillary pressure, relative permeability, oil viscosity and initial water saturation [3]. Beckner et al. (1987) modeled the imbibition process as a diffusion process. He assumed that diffusion coefficient is nonlinear and also gravity forces are ignored [4]. Behbahani et al. (2006) simulated spontaneous countercurrent imbibition in one and two-dimensional systems. The simulated results matched the experimental results reported by several authors, and they concluded that the conventional Darcy's law for multiphase flow was adequate to describe spontaneous countercurrent imbibition [2].

In this study, three-dimensional model is presented for study of spontaneous imbibition process. The two-phase mass balance equations for water and oil using Darcy's equation and other auxiliary equations are solved simultaneously by finite difference method. Then, this model is used to study the effect of gravity forces on imbibition performance.

2. MATHEMATICAL FORMULATION

A mathematical formulation of the spontaneously imbibition process is presented here. It is assumed that the fluids and core are incompressible. The model consists of two main equations containing of conservation mass equation for water and oil. Conservation mass equation for each phase (oil and water) in x, y and z directions in a cube core (Fig.1) is obtained as below:

$$\emptyset\frac{\partial(S_w)}{\partial t}+\frac{\partial(u_{wx})}{\partial x}+\frac{\partial(u_{wy})}{\partial y}+\frac{\partial(u_{wz})}{\partial z}=0 \qquad (1)$$

$$\emptyset\frac{\partial(S_o)}{\partial t}+\frac{\partial(u_{ox})}{\partial x}+\frac{\partial(u_{oy})}{\partial y}+\frac{\partial(u_{oz})}{\partial z}=0 \qquad (2)$$

Water and oil velocities can be obtained using Darcy's law as follow:

$$u_{hx}=\frac{-k_h}{\mu_h}\left(\frac{\partial P_h}{\partial x}-\rho g_x\right) \qquad (3)$$

The total velocity for counter-current imbibition process is zero:

$$u_t=0 \qquad (4)$$

$$u_{ox}+u_{wx}=0 \qquad (5)$$

by inserting Darcy's equation in the Eq. (5):

$$\frac{-ko}{\mu o}\left(\frac{\partial P_o}{\partial x}-\rho g_x\right)+\frac{-kw}{\mu w}\left(\frac{\partial P_w}{\partial x}-\rho g_x\right)=0 \qquad (6)$$

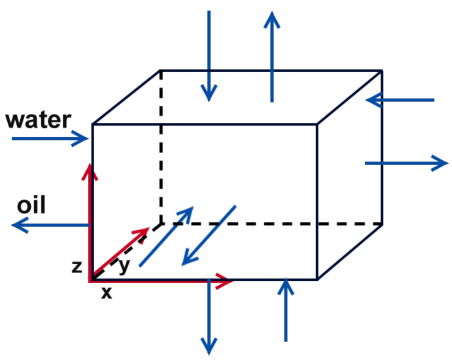

Figure. 1 Core geometry for the spontaneous imbibition when all faces are open

Auxiliary equations are the correlation between capillary pressure and water saturation that can be used to replace the capillary pressure by saturation equations as follows:

$$P_{cw} = P_o - P_w = f(S_w) \tag{7}$$

Where,

$$\frac{\partial P_o}{\partial x} = \frac{\partial P_c}{\partial x} + \frac{\partial P_w}{\partial x} \tag{8}$$

By inserting Eq. (8) into Eq. (6):

$$\frac{\partial Pw}{\partial x} = -\frac{g_x\left(\frac{k_w}{\mu_w}\rho_w - \frac{k_o}{\mu_o}\rho_o\right) + \frac{k_o}{\mu_o}\frac{\partial Pc}{\partial x}}{\frac{k_o}{\mu_o} + \frac{k_w}{\mu_w}} \tag{9}$$

By substituting Eq. (9) into the Darcy's equation, the oil and water velocities are obtained as function of capillary pressure as follow:

$$u_{wx} = \frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\left(g_x(\rho_w - \rho_o) + \frac{\partial Pc}{\partial x}\right) \tag{10}$$

$$u_{wy} = \frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\left(g_y(\rho_w - \rho_o) + \frac{\partial Pc}{\partial y}\right) \tag{11}$$

$$u_{wx} = \frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\left(g_z(\rho_w - \rho_o) + \frac{\partial Pc}{\partial z}\right) \tag{12}$$

Finally, by inserting above equations into Eq. (1), the governing equation for internal blocks will be obtained as follows:

$$\varphi\frac{\partial(sw)}{\partial t} + \frac{\partial\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\left(g_x(\rho_w - \rho_o) + \frac{\partial Pc}{\partial x}\right)\right)}{\partial x} + \frac{\partial\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\left(g_y(\rho_w - \rho_o) + \frac{\partial Pc}{\partial y}\right)\right)}{\partial y} + \frac{\partial\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\left(g_z(\rho_w - \rho_o) + \frac{\partial Pc}{\partial z}\right)\right)}{\partial z} = 0 \tag{13}$$

The capillary diffusivity coefficient (CDC) is defined as:

$$D(S_w) = -\frac{Kk_{ro}}{\mu_o}\frac{1}{1 + \frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}}\frac{dP_c}{dS_w} \tag{14}$$

D is a non-linear function of water saturation. We define the normalized water saturation as below:

$$S = f(x, y, z, t) = \frac{S_w - S_{wi}}{1 - S_{or} - S_{wi}} \tag{15}$$

So, S is the ratio of recovered oil to the total recoverable oil. By inserting the parameter S, Eq. (13) converts into the following correlation:

$$\varphi\frac{\partial(S)}{\partial t} + \frac{\partial}{\partial x}\left(D.\frac{\partial S}{\partial x}\right) + \frac{\Delta\rho.g_x}{1 - S_{wi} - S_{or}}.\frac{\partial}{\partial S}\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\right).\frac{\partial S}{\partial x} + \frac{\partial}{\partial y}\left(D.\frac{\partial S}{\partial y}\right) + \frac{\Delta\rho.g_y}{1 - S_{wi} - S_{or}}.\frac{\partial}{\partial S}\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\right).\frac{\partial S}{\partial y} + \frac{\partial}{\partial z}\left(D.\frac{\partial S}{\partial z}\right) + \frac{\Delta\rho.g_z}{1 - S_{wi} - S_{or}}.\frac{\partial}{\partial S}\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\right).\frac{\partial S}{\partial z} = 0 \tag{16}$$

In order to investigate the effect of gravity, a cube core has been modeled in z direction (Fig.2), in this state, all surfaces except one are impermeable. So, Eq. (16) converts into the following correlation:

$$\varphi\frac{\partial(S)}{\partial t} + \frac{\partial}{\partial z}\left(D.\frac{\partial S}{\partial z}\right) + \frac{\Delta\rho.g_z}{1 - S_{wi} - S_{or}}.\frac{\partial}{\partial S}\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\right).\frac{\partial S}{\partial z} = 0 \tag{17}$$

Eq. (17) is discretized as following:

$$S(i,j,k)^{n+1} - S(i,j,k)^n + \frac{Di,j,k+\frac{1}{2}.\Delta t}{\varphi.h^2}.S(i,j,k+1)^{n+1} - \frac{Di,j,k+1/2.\Delta t}{\varphi.h^2}.S(i,j,k)^{n+1} + \frac{Di,j,k-1/2.\Delta t}{\varphi.h^2}.S(i,j,k-1)^{n+1} - \frac{Di,j,k-1/2.\Delta t}{\varphi.h^2}.S(i,j,k)^{n+1} + \frac{\Delta t}{\varphi.2h}.\frac{\Delta\rho.g_z}{1 - S_{wi} - S_{or}}.\frac{\partial}{\partial S}\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\right).S(i,j,k+1)^{n+1} - \frac{\Delta t}{\varphi.2h}.\frac{\Delta\rho.g_z}{1 - S_{wi} - S_{or}}.\frac{\partial}{\partial S}\left(\frac{k_w k_o}{k_o\mu_w + k_w\mu_o}\right).S(i,j,k-1)^{n+1} = 0 \tag{18}$$

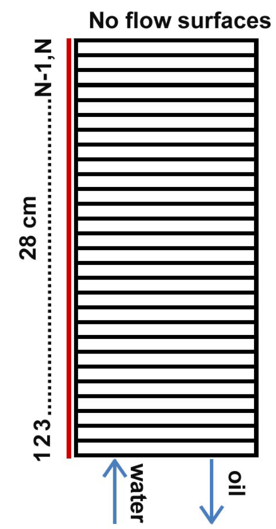

Figure. 2 Grid geometry for 1D

u_{wx} u_{wz} The superscript indicates the time level. n is the old time level for which we have a complete solution and all variables and properties. n+1 is the new time level and variables and properties are unknown. Eq. (18) is valid for the internal blocks.

At the permeable surfaces in contact with water, boundary conditions are set to be S_w=1 - S_{or}, the blocks of impermeable surfaces are unknown so the mass balance must be written for impermeable surfaces. Initial conditions are equal to initial water saturation, so S=0.

At the impermeable surface (In node number N), mass equation is given as below:

$$\frac{\frac{2k_wk_o}{k_o\mu_w+k_w\mu_o}\left(g(\rho_w-\rho_o)+\frac{\partial Pc}{\partial S_w}.\frac{\partial S_w}{\partial z}\right)}{dz}=\varphi\frac{\partial S_w}{\partial t} \tag{19}$$

Eq. (19) is discretized as below:

$$\frac{2k_wk_og(\rho_w-\rho_o)}{(k_o\mu_w+k_w\mu_o)h_z.(1-S_{wi}-S_{or})}+\frac{2D}{h_z}.\frac{S(i,J,N)^{n+1}-S(i,j,N-1)^{n+1}}{h_z}=\varphi.\frac{S(i,j,N)^{n+1}-S(i,j,N)^{\mathbf{n}}}{\Delta t} \tag{20}$$

Finally, there are (N-1) equations for (N-1) unknown nodes that must be solved simultaneously.

If the matrix is very low hydrophilic or the core is too high, the capillary pressure has low effect on imbibition process. In this section, a mathematical formulation of the imbibition is presented, that the capillary terms are neglected. Water and oil velocity in z direction can be obtained using Darcy's law, so following equations will be obtained:

$$u_{wz}=\frac{K.k_{rw}}{\mu_w}\left(-\frac{\partial P_{wo}}{\partial z}-\rho_wg_z\right) \tag{21}$$

$$u_{oz}=\frac{K.k_{ro}}{\mu_o}\left(-\frac{\partial P_{wo}}{\partial z}-\rho_og_z\right) \tag{22}$$

$$\frac{\partial P_w}{\partial z}=-\frac{\mu_wU_{wz}}{Kk_{rw}}-\rho_wg_z \tag{23}$$

$$\frac{\partial P_o}{\partial z}=-\frac{\mu_oU_{oz}}{Kk_{ro}}-\rho_og_z \tag{24}$$

$$\frac{\partial P_c}{\partial z}=\frac{\partial(P_o-P_w)}{\partial z}=\frac{\mu_wu_{wz}}{Kk_{rw}}-\frac{\mu_ou_{oz}}{Kk_{ro}}-\Delta\rho g_z \tag{25}$$

$$\frac{\partial P_c}{\partial z}=0 \tag{26}$$

$$u_{oz}=-u_{wz} \tag{27}$$

$$u_{wz}=\Delta\rho.g.K\left(\frac{k_{ro}k_{rw}}{k_{ro}\mu_w+k_{rw}\mu_o}\right) \tag{28}$$

by inserting Eq. (28) into Eq. (1), the governing equation will be obtained as follows:

$$\varphi\frac{\partial S}{\partial t}+\frac{\partial}{\partial S_w}\left(\Delta\rho.g_z.K\frac{k_{ro}k_{rw}}{k_{ro}\mu_w+k_{rw}\mu_o}\right)\frac{\partial S}{\partial z}=0 \tag{29}$$

Eq. (29) is discretized as below:

$$\frac{S(i,j,k)^{n+1}-S(i,j,k)^n}{\Delta t}+\frac{\partial}{\partial S_w}\left(\Delta\rho.g_z.K\frac{k_{ro}k_{rw}}{k_{ro}\mu_w+k_{rw}\mu_o}\right),\frac{S(i,j,k+1)^{n+1}-S(i,j,k-1)^{n+1}}{2h_z}=0 \tag{30}$$

Equation (30) is valid for the internal blocks. In node number N, the mass equation must be written again, mass equation is given as below:

$$\frac{\frac{2k_wk_o}{k_o\mu_w+k_w\mu_o}(g(\rho_w-\rho_o)}{dz(1-S_{or}-S_{wi})}=\varphi\frac{\partial S}{\partial t} \tag{31}$$

eq. (31) is discretized as below:

$$\frac{\frac{2k_wk_o}{k_o\mu_w+k_w\mu_o}(g(\rho_w-\rho_o)}{dz(1-S_{or}-S_{wi})}=\varphi\frac{S(i,j,N)^{n+1}-S(i,j,N)^n}{\Delta t} \tag{32}$$

Finally, there are (N-1) equations for (N-1) unknown nodes. These equations must be solved simultaneously. MATLAB program is used to simulate, and fig.3 shows the modeling results.

3. RESULTS AND DISCUSSION

In order to investigate the sensitivity of gravity on counter-current imbibition rate, the vertical cube core was modeled by finite difference method and using MATLAB program.

The numerical results are compared with the experimental data of oil recovery and simulation results by Eclipse®-100, in Fig.3 to examine the validity of the proposed method [2, 5].

Fig.3 shows that numerical and experimental results are consistent, except at the end times. The capillary or gravity forces can act as dominant force to control the imbibition rate.

According to the matrix property and the position of the matrix and the fracture, the dominant force is determined. In the most position the capillary forces are driving force and the gravity effect can be neglected. Fig.3 and 4 show oil recovery as a function of time for three different positions that driving force is only capillary force, only gravity force or both of them. Fig.3 shows oil recovery in a core with low height, h=0.28m, here driving force is capillary pressure and gravity force has no significant effect.

Fig.4 shows oil recovery in a core with h=3.0 m, in this height, the effect of gravity is significant, also, the effect of gravity is more than capillary effects and both of them are driving force. So, increasing the height of core will increase the effect of gravity and decrease the effect of capillary forces. Neglecting the gravity force will cause a lot of error in predicting the oil recovery by imbibition process from high reservoirs.

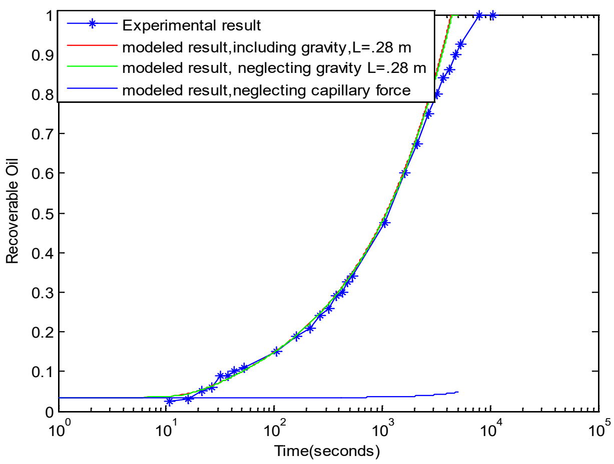

Figure. 3 Comparison of experimental and simulated results for three different positions that driving force is only capillary force, only gravity force or both of them, the core height=0.28m. The experimental data is from Bourbiaux et al. [5] and Behbahani et al. [2].

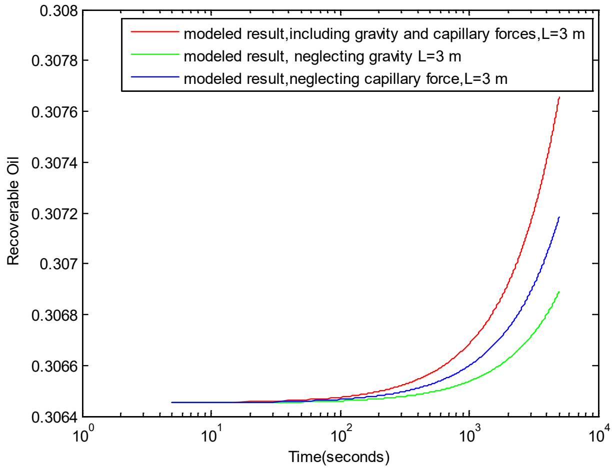

Figure. 4 Comparison of oil recovery as a function of time for three different positions that driving force is only capillary force, only gravity force or both of them, the core height=3m.

4. CONCLUSIONS

- The oil recovery mechanism during imbibition in cube core has been analyzed using the conservation mass and auxiliary equations.
- The capillary force is driving force of imbibing the water into the matrix block with low height.
- The effect of gravity force is significant for oil recovery from high matrix.
- Increasing the height of core will increase the effect of gravity and decrease the effect of capillary forces.

5. NOMENCLATURE

CDC : Capillary diffusivity coefficient

D : Capillary diffusion coefficient [m^2/s]

H : fluid [oil or water]

i : Integer denoting cell location in x-directions

j : Integer denoting cell location in y-directions

k : Integer denoting cell location in z-directions

K : Absolute permeability [darcy or m^2]

k_h : Effective permeability to fluid h [darcy or m^2]

k_{ro} : Relative permeability to oil

k_{rw} : Relative permeability to water

n : Integer indicating time level

N : Total number of nodes

P_c : Capillary pressure [Pa]

P_h : fluid h (oil or water) pressure [Pa]

P_o : Oil pressure [Pa]

P_w : Water pressure [Pa]

S : Normalized water saturation [fraction, m^3/m^3]

S_{Ai} : Distance from the open surface to the center of the matrix [m]

S_o : Oil saturation at time t in position (r,z) [fraction,m^3/m^3]

S_{or} : Residual oil saturation in the matrix [fraction, m^3/m^3]

S_w : Water saturation at time t in position (r,z) [fraction,m^3/m^3]

S_{wi} : Initial water saturation in the matrix [fraction,m^3/m^3]

T : Imbibition time [s]

u_{hx} : Fluidh (oil or water) velocity in x-direction [m/s]

u_{ox} : Oil velocity in x-direction [m/s]

u_{oy} : Oil velocity in y-direction (m/s)

u_{oz} : Oil velocity in z-direction [m/s]

u_t : Total velocity [m/s]

u_{wx} : Water velocity in x direction [m/s]

u_{wy} : Water velocity in y direction [m/s]

u_{wz} : Water velocity in z direction [m/s]

V : Volume of the matrix [m^3]

x : Coordinate in x-direction

y : Coordinate in y-direction

z : Coordinate in z-direction

μ_h : Fluid h viscosity [Pa.s]

μ_o : Oil viscosity [Pa.s]

μ_w : Water viscosity [Pa.s]

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آنالیز فرآیند آشام غیر همسو از طریق حل مساله با روش تفاضل محدود و با در نظر گرفتن نیروی گرانش

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چکیـــــده

فرآیند آشام خود به خودی غیرهمسو یکی از مهمترین روش های برداشت نفت از مخازن شکاف دار آبدوست با تراوایی پایین بلوک ماتریکس به حساب می آید. این مطالعه بر آن است که از طریق مدلسازی عددی، فرآیند آشام را که در آن آب از طریق نیروی های گرانش و موییتگی نفت موجود در بستر مکعبی شکل را جاروب می کند را بررسی کند و اثر نیروی گرانش بر عملکرد این فرآیند را معین کند. در این تحقیق، پدیده ی آشام به عنوان فرآیند نفوذی در نظر گرفته شده است. روش عددی تفاضل محدود ضمنی برای حل معادلات حاکم بر فرآیند آشام خودبه خودی به کار گرفته شده است. دقت مدل ارائه شده با صحه سنجی و مقایسه خروجی این مدل ریاضی و داده های آزمایشگاهی بررسی شده است.

واژگان کلیدی: آشام خود به خودی، آشام غیرهمسو، برداشت نفت، نیروی گرانش.